Invited Lecture

Research on Discussion in Mathematics Teaching: A Review of Literature From 2000 to 2020

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ABSTRACT For decades, reformers have emphasized discussion over recitation and lecture. Yet, traditional communication patterns are still dominant in mathematics classrooms internationally. In an effort to better understand this challenge, the present study investigates patterns and contributions of research on discussion in mathematics teaching. Based on systematic search in the Eric database, and in selected journals of mathematics education, 72 studies were reviewed. Based on analysis and discussion of the reviewed studies, it is suggested to develop conceptual clarity and include definitions of core terms like discussion, to consider alternative methods for studying discussion in teaching, and to consider shifting the focus from teacher actions to the entailments of the work of leading mathematical discussions.

Keywords: Discussion; Teaching; Mathematics; Literature review.

1. Introduction

The idea of discussion in teaching is not new. It is associated with the dialectic principles of the Socratic dialogues (cf. Sattler, 1943), Dewey's (1916) thinking about participation in a democratic society, and more. In an early textbook on social psychology, Ross (1908) stated that, "It is coming to be recognized that there is nothing of concern to human beings which may not profitably be discussed in the right spirit, by the right persons, at the right time" (p. 309). Later, Schwab (1954) described discussion as "indispensable to a good liberal education" (p. 51), and Cockroft (1982) listed discussion as a core element of mathematics teaching. Where traditional teaching involves communication formats like recitation, lecture, and teacher explanation, reform pedagogies often involve exploration and discussion (Smith, 1996).

A simplistic view of discussion in teaching is that the teacher should avoid telling the students, and instead step aside and let students discuss. In this sense, teaching by discussion would seem to involve less effort and a less prominent role of the mathematics teacher. Chazan and Ball (1999) were among the early critics of such a view. From analysis of two episodes of discussion, they unpacked the complexities and challenges in the role of the teacher in discussion and examined moves that teachers

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could make to moderate discussions. In another study, Lampert (2001) unpacked how the work of leading discussions consists of numerous problems that teachers must solve. Many of these problems involve navigation of seemingly unsolvable dilemmas, like "simultaneously teaching individual students and engaging the group as a whole in worthwhile activity," and "keeping the discussion on track while also allowing students to make spontaneous contributions that they considered to be relevant" (p. 174). More recently, leading discussions has been described as a core practice of mathematics teaching (Jacobs and Spangler, 2017), and volumes have been written to help teachers carry out this practice (e.g., Chapin et al., 2009; Kazemi and Hintz, 2014). Although books like these, along with frameworks like the "five practices" (Stein et al., 2008), have provided teachers with useful tools to support their planning, initiation, and orchestration of mathematical discussions, research still indicates that change in practice is slow, and that traditional teaching is still dominant. Everyone seems to agree about the need for change, and about the direction for such change, but the field has still not managed to change practice in this direction. This is a challenge. Being faced with a challenge like this, it seems logical to carefully consider previous efforts to approach it. I was therefore surprised to find that there were few reviews of research on discussion in mathematics teaching. There have been some reviews of research on discourse in mathematics education (e.g., Ryve, 2011), and Jacobs and Spangler (2017) provided a useful review of research on core practices, where leading discussions was one of two core practices they considered in detail, but I have not been able to find any comprehensive reviews of research on discussion in mathematics teaching. To mitigate this, the present study investigates what characterizes recent research on discussion in mathematics teaching. The aim of the study is to uncover trends in research, consider what has been emphasized and not, and thereby initiate professional deliberation about limitations and potential shifts in research on discussion in mathematics teaching. The following research questions are considered:

1. What are the core problems in studies of discussion in mathematics teaching?
2. What aspects of discussion are focused on in studies?
3. What methods are used for studying discussion in mathematics teaching?
4. What common reference literature can be identified in studies of discussion in mathematics teaching?

The study has been organized as a review of research on discussion in mathematics teaching, focusing on the last twenty years (2000–2020). Before presenting methods and results from the literature review, I elaborate on key terms and conceptual underpinnings.

2. Conceptual Background

The first key term to elaborate on is that of *discussion*. What do we mean by discussion? On the one hand, discussion is common term that is used frequently in everyday speech, and we "discuss" the weather, the news, last night's TV show, or a recent sports event.

On the other hand, discussion can refer to a more specific form of communication where we investigate or examine a complex issue to reach a solution.

The word "discuss" originates from Latin and is composed by two parts. The first part, "dis," means apart, and the last part, "quatere," means to shake. Etymologically, then, discussion means to shake something apart. If we confer with a contemporary dictionary, like the Oxford English Dictionary, discussion is defined like this:

> Treatment of a subject, in speech or writing, in which the various facts, opinions, and issues relating to it are considered; the action or process of talking about something in order to reach a decision or to exchange ideas (Discussion, n.d.)

A few things are worth noticing about this definition. First, discussion is always about *something*. There must be some subject or issue to be discussed. Discussions can be verbal or written, and written discussions can be synchronous or asynchronous, like in an online discussion forum. This study focuses on verbal discussions that take place synchronously in the context of the mathematics classroom. Second, discussion is "the action or process of talking about something in order to reach a decision or to exchange ideas." This indicates that discussion always has a *purpose*. Dillon (1994, p. 8) brings all these aspects together in the following definition:

> Discussion is a particular form of group interaction where members join together in addressing a question of common concern, exchanging and examining different views to form their answer, enhancing their knowledge or understanding, their appreciation or judgement, their decision, resolution or action over the matter at issue.

At least four perspectives are worth highlighting from this definition. First, Dillon describes discussion as a *particular* form of group interaction, and he makes a clear distinction between discussion and recitation. In recitation, the teacher typically asks a question, a student responds, and the teacher then evaluates their response; this is often referred to as an IRE pattern of communication. Discussions typically follow a different pattern of communication, where students often ask questions — not only to the teacher, but also to other students — and teachers do not always provide an evaluation of students input, but they might instead prompt students to comment on each other's thinking. Second, the emphasis on the question of common concern is useful to keep in mind. In a discussion, there must be a particular question or problem that a group wants to solve. In the classroom, the group typically consists of a teacher and their students. Third, there must be an exchange of ideas or views in a discussion; it is not sufficient to have a contribution from only one person. Different views must be exchanged and examined to constitute a discussion. Fourth, there might be different purposes of a discussion. Some discussions aim at enhancing knowledge or understanding, whereas other discussions aim at reaching a decision that might lead to some action.

The next key term in this study is "teaching." Studies of discussion in teaching necessarily involve a conception of teaching. Like discussion, the words "teach," or

"teaching" can be used in different ways in everyday speech. The Oxford English Dictionary defines teaching as "the imparting of instruction or knowledge; the occupation or function of a teacher" (Teaching, n.d.). Two things are worth noticing about this definition. First, teaching is always about something; there must be some content or knowledge at play. Second, teaching can also refer to the occupation or function of a teacher. Teaching is thus not only about actions that teachers perform, but it also refers to an occupation or professional practice.

The research literature applies different definitions of teaching, and these definitions relate to the dictionary definition referred to above. Many define teaching as the activities that are carried out by teachers. As an example, Gage (1978, p. 14) defined teaching like this: "By teaching I mean any activity on the part of one person intended to facilitate learning on the part of another". One interesting aspect of this definition is the focus on activity. Researchers, like Gage, who apply a process-product paradigm for studying teaching, often consider teaching as something teachers do, or activities performed by teachers. Another interesting aspect of the definition is the implied relationship between teaching and learning. Although teaching is the activity of one person, it has the intention of facilitating learning in another person. The definition seems to imply that teaching is something that teachers do, whereas learning is something students do. Although there is an intention of facilitating learning, teachers are dependent on someone else (the students) to be successful in their profession (Cohen, 2011).

Other researchers seem to attend more to the second aspect of the dictionary definition, when they define teaching as professional practice, or as work that needs to be done. For instance, Lampert (2010) describes teaching as a practice, and she frequently refers to the "work of teaching". Ball and Forzani (2009, p. 497) are in the same tradition when they define the work of teaching as "the core tasks that teachers must execute to help pupils learn". Their focus is more on identifying and understanding the tasks than on evaluating how particular teachers execute these tasks. This involves a shift in focus from considering teaching as something teachers do toward the tasks or core components of the work that teachers are faced with. The research literature describes these core components of the work in different ways. For instance, Lampert (2001) describes them as *problems* of teaching, indicating a metaphor of teaching as problem solving. Cohen (2011) describes the work of teaching by considering its *predicaments*; teachers are faced with numerous predicaments that they must deal with. Again, Ball and colleagues (2008) describe the core components of the work of teaching as *tasks of teaching* in their practice-based theory of mathematical knowledge for teaching (cf. Hoover et al., 2014). These are (mathematical) tasks that teachers are routinely faced with and must carry out in their work.

Considering teaching as work differs from the more conventional way of thinking about teaching as actions teachers perform. Studies that conceptualize teaching as something teachers do often focus on identifying patterns in teachers' actions or communication, or they attempt to evaluate the effectiveness of teachers' actions when

compared with some outcome variable. In contrast to this, consideration of teaching as work often leads to studies that aim at understanding what is involved in teaching (e.g., Ball, 2017), or developing a language to describe the core components of this work and pedagogies for learning or improving it (e.g., Boerst et al., 2011; Ghousseini, 2015).

3. The Literature Review

Selection of studies was carried out in two phases. The first phase involved manual searches in a selection of research journals in mathematics education. The following journals were included: *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, *The Journal of Mathematical Behavior*, *Mathematical Thinking and Learning*, *Mathematics Education Research Journal*, *ZDM*, *International Journal of Science and Mathematics Education*, and *Journal of Mathematics Teacher Education*. The first six were the same journals that Ryve (2011) considered in his review of research on discourse in mathematics education, but I decided to add the last two since they have become prominent in recent years.

Searching the archives of these journals for studies on discussion in mathematics teaching that were published between 2000 and 2020, 35 articles were included after initial screening and coding. Based on what was learned from the coding of these articles, a second phase was initiated that included more systematic searches in the Eric database. From the first phase of review, I observed that relevant articles tended to have key words like *teaching*, *mathematics*, and *discussion* in the title or abstract. A search for peer-reviewed journal articles in English, with these keywords — teaching, mathematics, and discussion — as search terms in the title or abstract, gave 146 articles. After initial screening and coding, and after deleting duplicates from the first phase, 37 additional articles were included in the review. Altogether, a total of 72 studies were included from the two phases of the literature review.

In both review phases, studies were excluded from the review if they were 1) not empirical (e.g., theoretical articles or review articles), 2) not about discussion, or 3) not about mathematics (e.g., some articles in the second phase focused on discussion in other subjects).

To answer the first research question, studies were coded in terms of:

- focus of the study
- how (much) the study emphasized discussion
- problem of the study (generic problem that was approached in the study)

To answer the second research question, concerning what aspects of discussion were focused on, studies were coded according to the following perspectives:

- definition (if the study provided explicit definition of discussion)
- phase (what phase in the work of leading discussions that was emphasized)
- talk moves (if the study included emphasis on talk moves or similar)
- norms (if the study included emphasis on establishing norms for discussion)

- demands (if the study focused on knowledge demands of leading discussions)

The third research question concerned methods to study discussion, and the following aspects were considered to answer this question:

- participants (number of participants)
- level (e.g., primary, or middle school)
- teachers (e.g., future, beginning, or experienced teachers)
- setting (e.g., professional development or teacher education context)

To investigate the fourth research question, reference lists of all articles were scanned, and references that relate to discussion were identified. These references were counted and compared across the total set of articles included in the review.

To illustrate the coding of studies, I briefly describe the study by Langer-Osuna and Avalos (2015). This study initially came up in both searches, since it was published in one of the journals that was targeted in the first phase (ZDM), and since it had a clear focus on discussion in mathematics teaching in the abstract. The study focused on implementation of progressive classroom practices. The overall problem of the study was about "how teachers facilitate discussion." The primary focus in the analysis was on the orchestration of discussion, and talk moves were discussed, although the authors focused more on students' use of talk moves than on the teacher's use of talk moves as a tool to orchestrate the discussion. The authors mentioned that norms have been established in the classroom, but the study as such did not have an explicit focus on the establishment of norms, and the study did not focus on knowledge demands of the work of leading discussions. It was a small-scale study that analyzed data from the grade 4 classroom of one practicing mathematics teacher in the United States. The setting was professional development of in-service teachers. Although the authors defined various kinds of talk that might take place during discussions, they did not define the concept of discussion as such. When considering the reference literature used, this study frequently referred to literature on dialogic education, like Littleton and Mercer (2013).

4. Results

Below is a presentation and discussion of results from the analysis of the studies in response to the four research questions.

4.1. Problems of the studies

Specific research questions are likely to differ across studies, and they are thus difficult to compare directly. Instead of comparing the specific research questions of the studies, I tried to identify the more general or overarching problems that the studies seem to address (Tab. 1). This corresponds with the way Hoover et al. (2016) identified problems in their review of studies of mathematical knowledge for teaching.

Tab. 1. Problems of the studies

Problem	No. of studies
What contributes to/supports discussion	21
How teachers facilitate discussion	11
What contributes to student learning in discussion	11
How teachers attend to students in discussion	8
What contributes to development of discussion	7
What contributes to participation in discussion	5
What students experience or learn from discussion	3
What demands teachers are faced with in discussion	2

By considering the problem statements in the articles — this includes the specific research questions, but also the overall framing of the problem in the studies — inductive codes were developed to describe more generic types of problem statements. As an example, Hintz and Tyson (2015) presented two research questions in their article: "1. How do an elementary teacher and his or her students listen to each other during a mathematical discussion? 2. How does the teacher support students to listen as mathematical sense-makers?" (pp. 301–302). I considered the overall problem in their study to be "How teachers attend to students in discussion".

We notice here that 21 studies approached a generic problem of what contributes to or supports discussions. One subgroup of studies in this category investigated use of diverse types of technology to support discussion. For instance, Hensberry and colleagues (2015) investigated how simulations can provide a context that supports whole-class discussions, whereas Slavit (2002) explored how an electronic forum can support classroom discussions. Another subgroup of studies that focused on what contributes to or supports discussion investigated various kinds of tools or frameworks. For instance, Casa (2013) investigated how a "talk frame" can be used as a tool to support discussion in mathematics classrooms. In another study, Wu and colleagues (2009) explored use of graph organizers and the "mathematician's chair" as tools to support problem solving discussions in mathematics. A third subgroup of studies focused on how teachers' knowledge or beliefs might support discussions. For instance, Bray (2011) investigated how teachers' knowledge and beliefs influenced the way they handled student errors in classroom discussions, whereas Cengiz and colleagues (2011) studied how mathematical knowledge for teaching influenced teachers' instructional actions in discussions.

A second and related group of studies emphasized how teachers facilitate discussion. Many of these studies were small scale studies that investigated how one or a few teachers approach facilitation of classroom discussions in mathematics. Some studies involved attempts to try out unusual ways of organizing discussions, for instance by introducing random grouping of students (Carter, 2019). Other studies unpacked different components of the work of facilitating discussion. For instance, Selling (2016) explored what teacher moves that were used to make mathematical practices explicit for students, and what was made explicit about these practices. In another study, Zolkower and Shreyar (2007) described the moves a teacher made to

press students to express their mathematical thinking verbally in "thinking-aloud discussions".

A third group of studies focused on what contributes to student learning in discussion. Many studies analyzed what teacher actions that support student learning in discussions. An example is the study by Vale and colleagues (2019), who explored the leading of problem-solving discussions through lesson study. They found that teachers attend more productively to student responses — and can select appropriate student responses — when they have solved the problems for themselves first and engage in the practice of anticipating children's responses. In another study, da Ponte and Quaresma (2016) found that an appropriate level of challenge in problems was necessary to foster productive learning situations. In yet another study, Lim and colleagues (2020) found that teachers' use of follow-up questions can stimulate student learning and participation in discussions.

A fourth group of studies considered how teachers attend to students in discussion, which can be an aspect of the facilitation of discussion. Attending to students is closely related to teacher noticing, and Scherrer and Stein (2013) explored how an intervention influenced what teachers notice during classroom discussions. In another study, O'Connor (2001) investigated how a teacher's use of questions in discussion can stimulate students' thinking. In yet another study, Hintz and Tyson (2015) investigated the listening of teacher and students in classroom discussions. They highlighted "complex listening," which involves listening evaluatively, interpretively, as well as hermeneutically, and they argued that this way of listening is necessary to facilitate mathematical sense-making. For the teacher, this involves, among other things, to take a "listening stance" and be curious about students' thinking.

Other groups of studies focused on what contributes to development of (e.g., Aguirre and del Rosario Zavala, 2013), or participation in (e.g., Ing et al., 2015), mathematical discussions. A few studies investigated what students experience or learn from discussion (e.g., Gellert and Steinbring, 2014), and two studies focused on the demands that teachers are faced with in discussion (e.g., Leikin and Dinur, 2007). Finally, there were four categories that only had one study each.

4.2. Aspects of discussion in focus

Tab. 2. Aspects of discussion that are in focus

Focus in studies	No. of studies
Orchestration	37
Talk moves	18
Norms	10
Demands	10
Definition	7

More than half of the studies focused on the orchestration of discussion, and many of them also involved some focus on talk moves. Studies applied different notions of "moves" teachers can use to support discussions. Some referred to them as "teacher

moves," or "didactic moves," whereas others referred to a commonly known list of "talk moves" (Kazemi and Hintz, 2014). Whereas much emphasis has been placed on moves teachers can make as they talk in discussions, less emphasis has been made on the complexity of listening in discussions. This was the primary focus in the study by Hintz and Tyson (2015), who considered different forms of listening in conjunction with other kinds of moves teachers can make while leading mathematical discussions. Despite different definitions and terms, all these studies had some emphasis on moves teachers can or should make during discussions.

Ten studies focused on norms. Again, I was generous in my interpretations and included studies that only briefly mentioned norms although the study was not primarily about norms. Only a few studies had an explicit focus on norms or development of norms for discussion. For instance, Kline (2008) had a primary focus on establishing a classroom environment for discussion. Based on her experience from long-term professional development with teachers, she unpacked what needs to be considered "when establishing a tone that encourages children to think during whole-group discussions, including addressing children's diverse thinking approaches and using their incorrect solutions" (p. 145).

Another group of ten studies had a focus on knowledge demands that might be entailed in teaching with discussion. An example is the study by McCrone (2005), where a fifth-grade classroom was observed over a full year, focusing on how student contributions to discussions develop over time, and the challenges the teacher was faced with in this work. The analysis in this study also involved emphasis on negotiation of sociomathematical norms, like the common belief that the teacher is the one who has authority to decide whether proposed solutions are valid.

Finally, only seven out of 72 studies defined what they meant by discussion — again with a generous interpretation of what constitutes a definition. For instance, McCrone (2005) defined discussion as "one aspect of discourse, namely, to describe the nature of small group and whole group discussions centered on making sense of mathematics problems" (p. 112). A more concise definition was given by Tyminski et al. (2014) who referenced Pirie and Schwarzenberger (1988, p. 461), who defined a mathematical discussion to be "purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction." Tyminski et al. (2014, p. 465) also clarified that a "discussion can be called mathematical to the extent it contributes to students' mathematical understanding and reasoning." These were exceptions, however, as most studies did not define discussion, and many applied different interpretations of discussion in their studies.

4.3. Methods for studying discussion

Several aspects were considered in the analysis of methods for studying discussion, but the most striking difference was found when comparing the sample size of studies (Tab. 3).

Tab. 3. Sample size of studies

Sample size	No. of studies
Small scale (<10)	55
Medium 1 scale (10–29)	5
Medium 2 scale (30–70)	2
Large scale (70>)	6

We notice that 55 of the 72 studies were small scale studies with less than ten participants; many studies only focused on one teacher[2]. Three of the large-scale studies focused on students rather than on teachers and thereby had a larger sample size than if they had focused on the teachers. The emphasis on how students experience discussions was shared across these studies, and some also focused on how students learn from discussions.

This tendency of mostly small-scale studies with a qualitative design is not surprising, since most studies focused on various aspects of the interactions between teachers and students in discussion, and few, if any, instruments have been developed to measure aspects of discussions quantitatively.

The study by Bragg (2012) was one of the few large-scale studies, and the focus was on investigating how game playing might contribute to mathematical learning. The study measured the impact of an intervention by use of achievement tests. The participants ($n = 112$) were thus students. In another study, Ing et al. (2015) investigated students' participation in discussions, and again the sample consisted of students ($n = 71$). These students were from six classrooms, and the team spent six months observing the classrooms before initiating the formal phase of data collection. Lesson videos were coded with a particular emphasis on student participation — primarily in terms of student explanations and engagement with the ideas of other students — and teachers' support. The study also included a written post-test of students' thinking. In yet another large-scale study, Lemonidis and Kaiafa (2019) measured the effect of including storytelling strategies on students' learning of fractions, and they compared results from an experimental group and a control group (each with $n = 38$).

The study by Jackson et al. (2013) was among the few large-scale studies that focused on the teachers ($n = 165$), and this study explored the relationship between use of cognitively demanding tasks and students' opportunities to learn in discussions. These researchers video recorded mathematics lessons over two days with each of the participating teachers, which constituted a total of 460 lessons that were analyzed using "an expanded version of the Instructional Quality Assessment" (p. 658). This instrument was developed from the Mathematical Tasks Framework, and it targets the interactions between teacher and students in discussions. The instrument identifies opportunities to learn, but it does not measure student learning. This can be considered among the most significant studies in the review, and it provides an interesting example

[2] Four studies did not provide clear information about sample size, so therefore the total in table 3 only adds up to 68.

of a study that involved the use of frameworks to measure aspects of mathematical discussions.

Although some of the large-scale studies were impressive in size and scope, many of the small-scale studies also provided important contributions. For instance, O'Connor's (2001) study unpacked important aspects of leading mathematical discussion, with a particular emphasis on how teachers' use of questions in a position-driven discussion might support the development of students' mathematical thinking. This was one among several studies that illustrated how case studies of one teacher can also provide important insights into the work of leading mathematical discussions. Another example was the study by McCrone (2005). Whereas Ing et al. (2015) studied students' participation in discussion in a large-scale study, McCrone (2005) investigated the development of student participation in discussions by following a teacher and her grade 5 students over a year. Through a longitudinal qualitative study, she unpacked how the role of the teacher and her use of questions might stimulate student participation by supporting their development from non-active to active listening, and to draw upon other students' thinking.

4.4. Reference literature

In his review of research on discourse in mathematics education, Ryve (2011) identified several core theoretical and epistemological traditions that were referenced. When reviewing research on discussion in mathematics teaching, few theoretical frameworks or traditions emerged. Surprisingly, it was difficult to identify any core body of literature, and there was significant variation in the literature that was referenced across studies. Candidates for a core body of literature from before 2000 were:

- Ball's (1993) unpacking dilemmas of teaching elementary mathematics
- Lampert's (1990) study of altering roles in mathematics classroom discourse
- Stein et al. (1996) with their analysis of cognitive demands in mathematical tasks
- Yackel and Cobb (1996) on sociomathematical norms in discussions

Even though these were among the most frequently used references, each of them was only referenced in a few studies (5–10 studies). More recent candidates for a body of core literature on discussion (published after the year 2000) were:

- Chapin et al.'s (2009) sourcebook on classroom discussions in mathematics
- Kazemi and Hintz (2014) with their book on structuring and leading discussions
- Lampert's (2001) seminal work on teaching with problems
- Stein et al. (2008) and their five practices for orchestrating discussions

Again, the number of references to these more recent candidates for core literature was relatively small. It was also surprising to notice that many core references from

the general literature on discussion in education (e.g., Dillon, 1994) were mostly absent from the list of references in studies that were included in the review.

5. Concluding discussion

Based on the present review of research on discussion in mathematics teaching, I will highlight three issues that are worth attending to in research on discussion in mathematics teaching. With each of these issues, I will point at limitations of research and suggest efforts to progress.

The first issue revolves around conceptual clarity. Dillon (1994) noticed that there was confusion of terms in studies of discussion in education, and he stressed the importance of distinguishing discussion from other types of interactions and providing clear definitions. Similarly, in his review of research on discourse in mathematics education, Ryve (2011) found a lack of conceptual clarity, and he argued that this might threaten the cumulative nature of research. The present review shows that few studies of discussion in mathematics teaching define what they mean by discussion, and studies tend to use the term in diverse ways. This is a significant challenge for our field. Coherence of terms might not be a requirement in research, but clarity is. If studies fail to clarify what they mean by the core constructs they investigate — like discussion — successive studies will be hard pressed to build on them. Ryve (2011) argued that this was critical for research on discourse, and I argue that this is equally important in research on discussion in mathematics teaching. An everyday concept like discussion might be particularly elusive in this respect, since everyone uses it, and everyone thinks they know what it means.

A second issue relates to methods for studying discussion in mathematics teaching. It is interesting to notice that most studies of discussion in teaching are small-scale, qualitative case studies. These studies provide illustrations of what discussions might look like, and they explore various aspects of discussions — often providing existence proofs. This tendency might be related to a general lack of instruments to measure important aspects of discussion in mathematics teaching. In one of the few quantitative studies, Jackson et al. (2013) applied an adapted version of the Instructional Quality Assessment, and this is one candidate measure for use in research on discussion in mathematics teaching. In their review of research on the core practice of leading classroom discussions, Jacobs and Spangler (2017) also noted that most studies were inductive case studies, and they suggested that development and use of observation instruments can be a promising method for studying discussion in mathematics teaching. Similarly, in their review of research on mathematical knowledge for teaching, Hoover et al. (2016) emphasized measurement work — not simply use of measures in correlational studies and assessment of practice, but development of measures as tools that may contribute to conceptualization of core constructs that are studied.

Finally, a third issue in research on discussion in mathematics teaching relates to the underlying conception of teaching. Research on mathematics teaching builds on a

long history of research on teaching, where a process-product paradigm has been prevalent. Within this paradigm, teaching is defined as something teachers do to help students learn, and studies of teaching have often considered process variables concerning teachers' actions or performance in relation to outcome variables, like some measure of student learning. Similarly, research on discussion tends to focus on the actions or moves teachers make and the underlying goals of these moves (Jacobs and Spangler, 2017). Focusing on teacher moves makes sense, in particular within a context of teacher education where the emphasis is on learning to lead discussions. Yet, it might be productive to shift toward a conception of teaching as professional work, where studies focus more on entailments of this work than on how teachers carry out the work. Ball (2017) and others have initiated a similar shift in research on mathematical knowledge for teaching, where the emphasis is on investigating problems, dilemmas, demands, or tasks that are entailed in the work of teaching, and this has laid the foundation for productive developments in this area of research (cf. Hoover et al., 2016). Research on discussion in mathematics teaching, however, still tends to emphasize actions by teachers or students in discussion, and the effectiveness of such actions. I suggest that a shift toward unpacking entailments of the work of leading mathematical discussions might stimulate further progress. Identifying demands of the complex work of leading discussions might lay the foundation for developing a professional practice that acknowledges the skills and knowing that are involved in leading mathematical discussions. One aspect of the work of leading discussion that might benefit from further research is the complex work of developing a classroom climate for discussion. Too many studies of discussion in mathematics teaching tend to investigate the orchestration of discussions in a context where such norms have already been established.

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